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INFLUENCE OF THE LINEARLY DISTRIBUTED CONCENTRATION OF CARRIERS ON THE OPERATING REGIMES OF A THERMOELECTRIC ARM

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A numerical solution of the boundary-value problem on calculation of the temperature field in a one-dimensional adiabatically insulated inhomogeneous thermoelectric arm has been obtained. The concentration gradient of nondegenerate carriers has been optimized, which enables one to substantially increase the thermoelectric Q-factor of the arm. The calculations have been carried out for two operating regimes — the regime of maximum temperature difference and that of maximum refrigerating capacity. The possible limits of optimization of the concentration gradient of charge carriers have been established.

The sharply increasing production of thermoelectric coolers has imposed growing requirements on the improvement of their efficiency. Therefore, the growth in the thermoelectric Q-factor is one of the most important problems of semiconductor materials science. The practical results achieved up to now are still very far from the theoretical limits of thermoelectric efficiency [1]. It is well known that one trend in improvement of the latter is the application of thermoelements inhomogeneous in length [2]. It has been established that in this case the thermoelectric efficiency is improved if the specific conductance grows and the thermoelectromotive force decreases from the hot end to the cold one. Such conclusions have been drawn in considering thermoelements with properties smoothly varying in length as the limiting case of a compound thermoelement [3]. Another approach based on solution of the boundary-value problem has been demonstrated in [4]. The variational problem posed by Ivanova and Rivkin was solved with the use of the Pontryagin maximum principle. This necessitated considerable simplifications of the initial conditions of the problem; in particular, it was assumed that the thermoelectric coefficient, the thermal conductivity, and the electrical conductivity weakly depend on temperature. However, the use of such a powerful optimization method in this case is not paid back by the simplifications made. Furthermore, the absence of systematic data in the work gives no way of drawing an unambiguous conclusion on selection of the optimum gradient of concentration of charge carriers. Finally, Ivanova and Rivkin considered just one regime — the regime of maximum temperature difference. In connection with what has been said above, we carried out numerical solution of the boundary-value problem and numerical optimization of the solutions obtained. The regimes of maximum temperature difference and maximum refrigerating capacity were considered. Since it was necessary to elucidate the efficiency of the action of the distributed Peltier effect, we disregarded the Thompson effect.

The temperature field of a one-dimensional isolated inhomogeneous thermoelectric arm in the steady-state regime without allowance for the Thompson effect is described by the steady-state heat-conduction equation

$$\frac{d}{dx}\left(\chi \frac{dT}{dx}\right) + \frac{y^2}{\sigma} - yT\frac{d\alpha}{dx} = 0 \tag{1}$$

with the boundary conditions

$$\chi \frac{dT}{dx}\bigg|_{x=0} = \alpha yT\big|_{x=0}, \quad T\big|_{x=1} = T_1, \tag{2}$$

the kinetic coefficients for the nondegenerate case have the form

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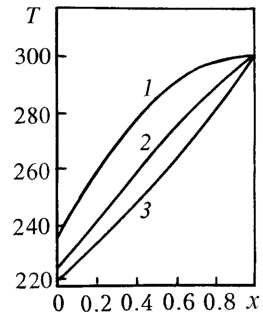


Fig. 1. Temperature distribution along the thermoelectric arm vs. factor g characterizing the impurity distribution in the arm: 1) g = 0; 2) 0.9; 3) 0.999.

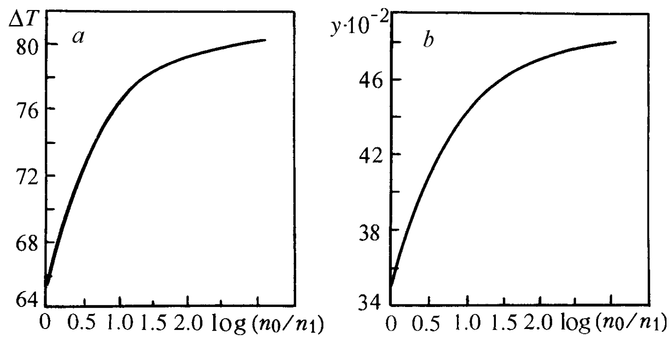


Fig. 2. Maximum temperature difference (a) and reduced current of the arm (b) vs. ratio of the concentrations of the carriers at the cold and hot ends of

$$\sigma = enu\,, \quad \chi = \chi_{\text{latt}} + 2\left(\frac{k}{e}\right)^2 T\sigma\,, \quad \alpha = \frac{k}{e}\left(2 + \ln\frac{2\,(2\pi mkT)^{3/2}}{nh^3}\right).$$

The mobility of the carriers u and the effective mass m were selected so that the thermoelectric properties corresponded to semiconductor materials with $Z = 3.0 \cdot 10^{-3}\ \text{K}^{-1}$ at $T_1 = 300$ K.

The parameter y can be called the reduced current. It is determined only by the physical properties of the substance and by the temperature and is independent of the geometry of the arm. The optimum value of the reduced current determines the value of the optimum current of the arm whose length and cross section are numerically equal. To find the optimum current of the arm with another geometry it suffices to multiply the reduced current by the ratio S/l.

Taking into account the dependence of the thermoelectromotive force on the concentration of the carriers (2) and disregarding the Thompson effect, we can write Eq. (1) of the boundary-value problem in the form

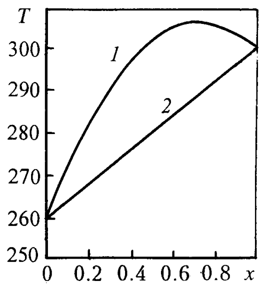
$$\frac{d}{dx}\left(\chi\frac{dT}{dx}\right) + \frac{y^2}{\sigma} + \frac{k}{e}\frac{yT}{n}\frac{dn}{dx} = 0\,. \tag{3}$$

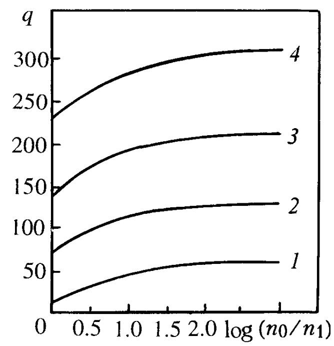
The concentration of the charge carriers along the arm is distributed by the linear law

$$n = n_0\,(1 - gx)\,. \tag{4}$$

In solving the boundary-value problem, we numerically optimized the temperature difference for the reduced current and the concentration n_0 at the cold end of the thermoelectric arm for a prescribed value of g. The range of variation of the quantity is $0 < g < 0.999$, which corresponds to the change in the ratio of the concentrations of the carriers at the cold and hot ends within $1 < \frac{n_0}{n_1} < 10^3$.

Results of the numerical solution of the boundary-value problem are presented in the figures. Figure 1 shows the temperature distribution along the thermoelectric arm for the optimum values of the reduced current in the regime





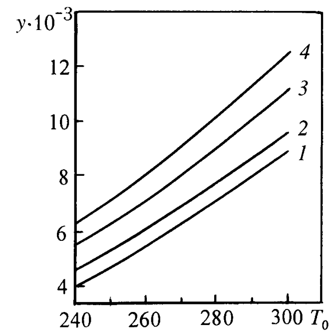


Fig. 3. Temperature distribution along the loaded arm vs. factor g: 1) g = 0; 2) 0.96.

Fig. 4. Reduced maximum refrigerating capacity vs. ratio of the concentrations of the carriers at the cold and hot ends of the arm for temperature differences of: 1) 60, 2) 40, 3) 20, and 4) 0 K.

Fig. 5. Reduced optimum current of the loaded arm vs. temperature of the cold end of the arm: 1) g = 0; 2) 0.5; 3) 0.9; 4) 0.999.

of maximum temperature difference. Curve 1 corresponds to a homogeneous arm (g = 0). In this case the maximum value of the temperature is attained at the point where the hot end of the arm is located. Below are the curves of temperature distribution in inhomogeneous arms with g = 0.9 and 0.999. In the case of an inhomogeneous density of the charge carriers the appearance of the distributed Peltier effect leads to a shift of the temperature maximum beyond the region in question. When g = 0.9, there is a curvature of the opposite sign. The presence of the gradient of concentration of the charge carriers causes a decrease in the temperature of the cold end of the arm due to the partial or total compensation for the Joule heat. In optimization for the concentration at the cold end of the arm, the optimum value of the concentration n_0 changes toward increasing. The growth in the optimum concentration of the charge carriers at the cold end decreases the value of the thermoelectromotive force at this point and hence is an unfavorable factor since it contributes to the decrease in the possible temperature difference. The dependence of the maximum temperature difference on the arm length on the logarithm of the ratio of the concentrations on the cold and hot junctions is presented in Fig. 2a. As is clear from the plot, it is inexpedient to change the concentration of the charge carriers by more than 8–10 times for the linear law of distribution, since additions to the temperature difference become very small for large ratios. Thus, a 10-fold change in the concentration enables us to increase the temperature difference by 17%, while a 1000-fold change yields an increase of 22%. Figure 2b plots the optimum values of the reduced current against the logarithm of the ratio of the concentrations on the cold and hot junctions. As is seen, large temperature differences require a larger current. The current grows by 26% with a 10-fold change in the concentration and by nearly 40% with a 1000-fold change.

From the practical viewpoint, of much greater interest is the regime of maximum refrigerating capacity. It is well known that a substantial overheating of the central region of the thermoelectric arm occurs in this regime; therefore, in this case the distributed Peltier effect can be of greater importance. For calculation in this regime we can represent the boundary condition at the cold end of the arm in the form

$$\chi \left.\frac{dT}{dx}\right|_{x=0} = \alpha y T \big|_{x=0} - q \,. \quad (5)$$

The absorption of the heat of the distributed Peltier effect will enable us to substantially decrease the overheating of the arm or to eliminate it completely. Figure 3 gives the plots of the temperature field in a loaded arm in the absence of the concentration gradient of the carriers and in the presence of it. It is seen that a 25-fold change in the concentration causes the compensation for the Joule heat as a result of which the temperature dependence becomes linear. Figure 4 gives the load characteristics of the arm for dissimilar temperature differences. As is seen, a 10-fold to 30-fold change in the concentration of the carriers accounts for the most significant growth of the reduced refrigerating capacity of the arm. For a temperature difference of 60 K the refrigerating capacity increases 3.3 times for a concentration change of 10 times and 4.1 times for a concentration change of 25 times. As the temperature difference decreases, the multiplicity of the increase in the refrigerating capacity decreases and attains values of 1.25 and 1.44 respectively for the zero temperature difference. Figure 5 shows the dependence of the optimum value of the reduced current on the temperature of the cold end for different concentration gradients. The plots differ from the linear dependences that occur when the differential thermoelectromotive force and the specific resistance are independent of the temperature.

Thus, the application of thermoelectric arms with a linear distribution of the concentration of the charge carriers along the arm length can be of interest for increasing the refrigerating capacity of thermoelectric refrigerators.

NOTATION

T, temperature of the thermoelectric arm as a function of the coordinate, K; χ, thermal conductivity, $W\cdot m\cdot K^{-1}$; σ, specific electrical conductivity, $\Omega^{-1}\cdot m^{-1}$; α, differential thermoelectromotive force, $V\cdot K^{-1}$; x, dimensionless coordinate, $0 < x < 1$; $y = Jl/S$, reduced current, $A\cdot m^{-1}$; l, length of the thermoelectric arm, m; S, cross-sectional area of the thermoelectric arm, m^2; T_1, temperature of the hot end of the thermoelectric arm. K; e, elementary charge, C; n, concentration of the charge carriers, m^{-3}; u, mobility of the charge carriers, $m^2\cdot V^{-1}\cdot sec^{-1}$; χ_{latt}, thermal conductivity of the crystal lattice, $W\cdot m\cdot K^{-1}$; k, Boltzmann constant, $J\cdot K^{-1}$; m, effective mass of the carriers, kg; h, Planck constant, J·sec; Z, parameter of thermoelectric efficiency, K^{-1}; g, proportionality factor; n_1, concentration of the carriers at the cold end of the thermoelectric arm, m^{-3}; $q = Ql/S$, reduced refrigerating capacity, $W\cdot m^{-1}$; Q, refrigerating capacity of the thermoelectric arm, W; T_0, temperature of the cold end of the thermoelectric arm, K. Subscripts: latt, lattice.

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